Finding collisions using differentials

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séminaire CASYS, Grenoble

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Radboud University

IN·DEI·NOMINE·FELICITER

Structure of this Talk

Question

Let (M, M') be a pair of public messages, then

$$\Pr[F(M) = F(M')]?$$

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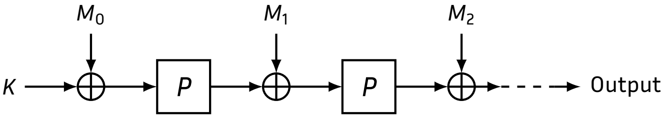
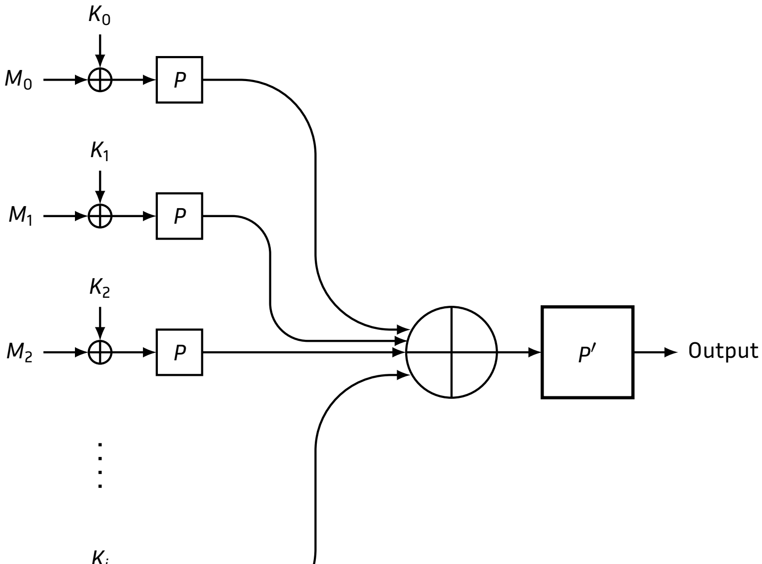
$$\Pr[F(M) = F(M')|M + M' = \Delta]?$$

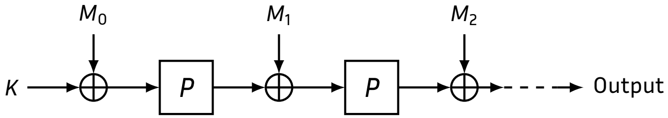
Serial Construction

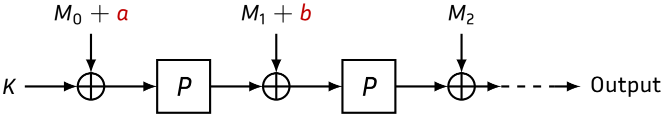


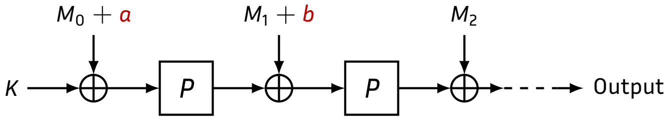
Parallel Construction



Plan of this Section


M_0
M_1
M_2
K
P
P
Output


$M_0 + a$
$M_1 + b$
M_2
K
P
P
Output



$$\Pr[Collision] = \mathsf{DP}(a, b)$$

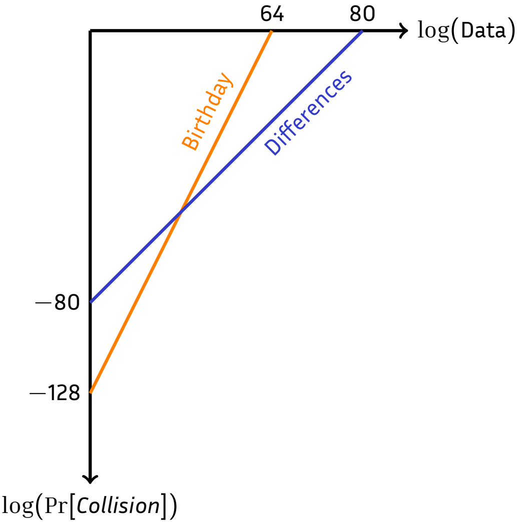
Framework

- P is "easier" than $P \circ P$.
- $\mathrm{DP}(a, b)$ is known for all $a, b \in \mathbb{F}_2^n$.

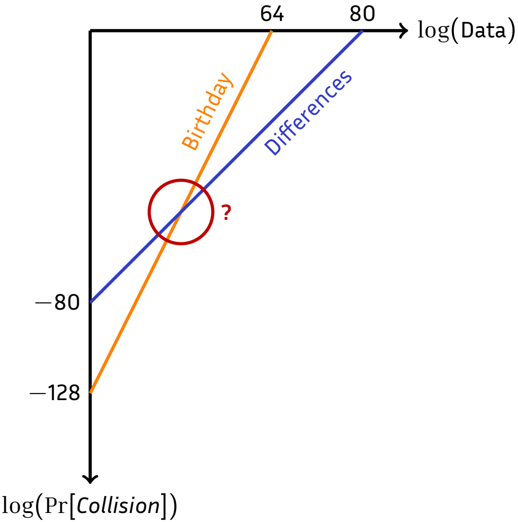
Goal

Find a collision in the output.

Birthday VS Difference



Birthday VS Difference



Not really Birthday

- (M_0, M_1),$(M_0 + a, M_1 + b)$,$(M'_0, M'_1), (M'_0 + a, M'_1 + b), \ldots$

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- (M_0, M_1),$(M_0 + a, M_1 + b)$,(M'_0, M'_1), $(M'_0 + a, M'_1 + b)$, ...
- $\Pr[Collision] = \delta_P(a, b)$

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Choose carefully the messages in a specific subspace...

Using Covering Vector spaces

$\langle (a_1, b_1), (a_2, b_2), \ldots, (a_v, b_v) \rangle = V$ such that

$$\mathrm{Moy}_V = \sum_{(a,b) \in V} \delta_{a,b} > \delta \,.$$

By making this strategy:

$$\begin{array}{c} M_0, M_1 \\ M_0 + a_1, M_1 + b_1 \\ M_0 + a_2, M_1 + b_2 \\ M_0 + a_1 + a_2, M_1 + b_1 + b_2 \\ \vdots \\ M_0 + \sum a_i, M_1 + \sum b_i \end{array}$$

$$\begin{array}{c} M'_0, M'_1 \\ M'_0 + a_1, M'_1 + b_1 \\ M'_0 + a_2, M'_1 + b_2 \\ M'_0 + a_1 + a_2, M'_1 + b_1 + b_2 \\ \vdots \\ M'_0 + \sum a_i, M'_1 + \sum b_i \end{array}$$

....

Using Covering Vector spaces

We get

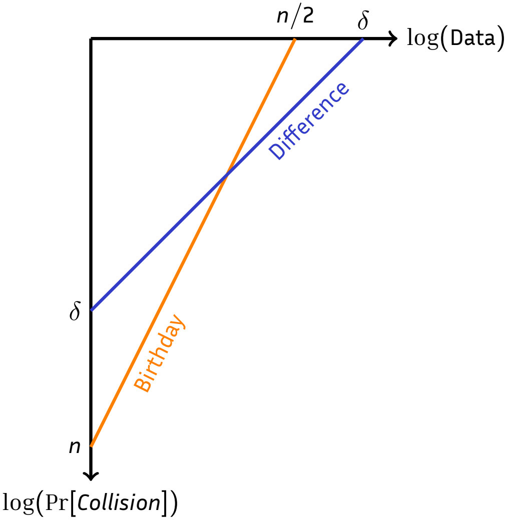
$$\Pr[Collision] = n_{block} \times \mathrm{Moy}_V \times 2^{v-1} + \frac{1}{2^n} \times \binom{n_{block}}{2} 2^{2v}$$

$$\Pr[Collision] = \frac{D}{2} \times \mathrm{Moy}_V + \frac{D^2}{2^n}$$

And

$$\mathrm{Moy}_V >> \delta\,??$$

What could possibly be wrong?



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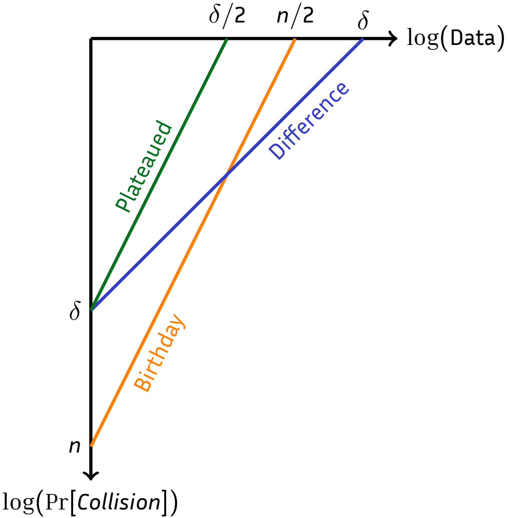

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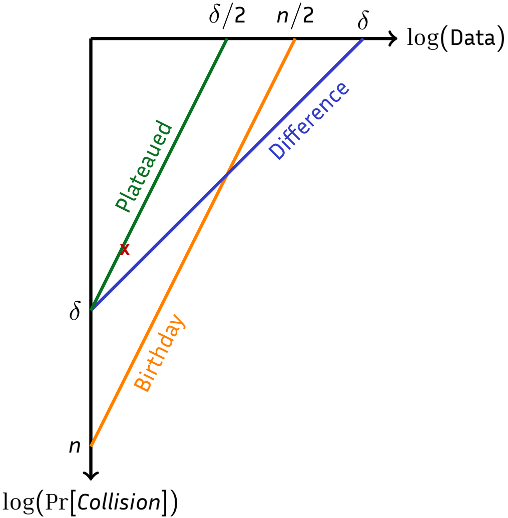

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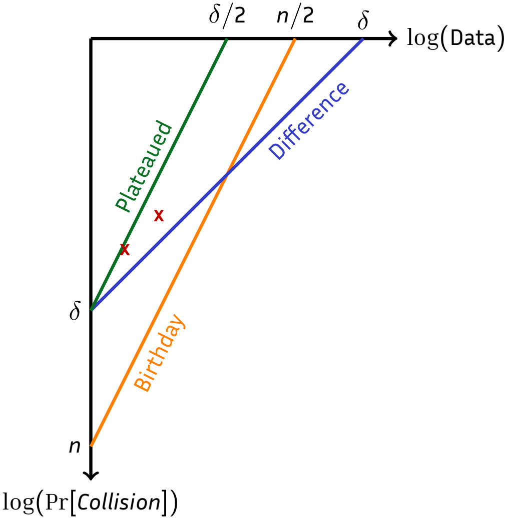

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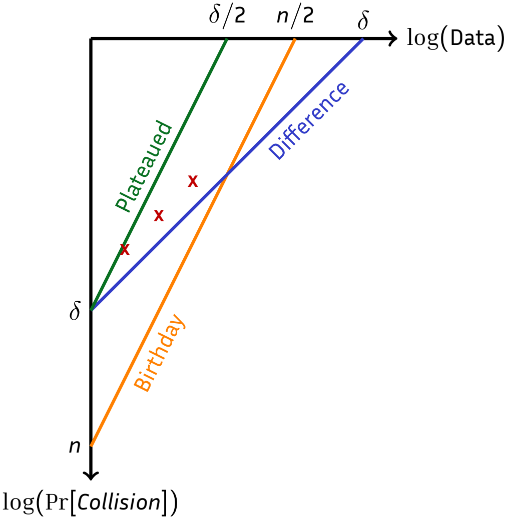

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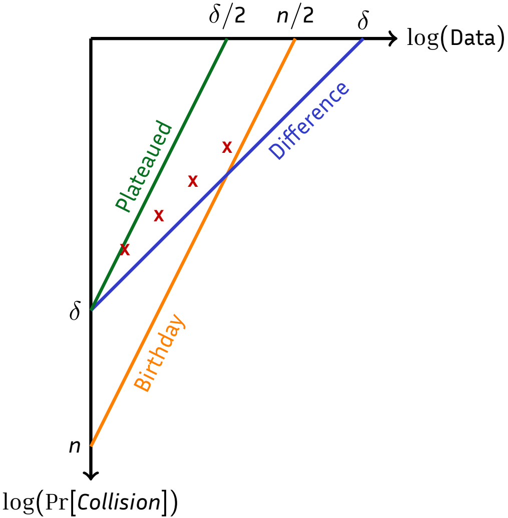

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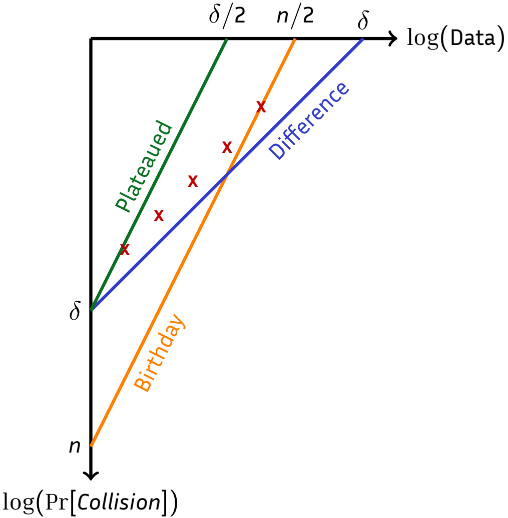

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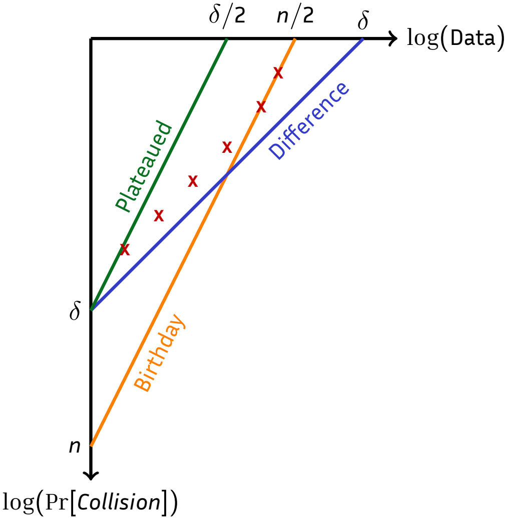

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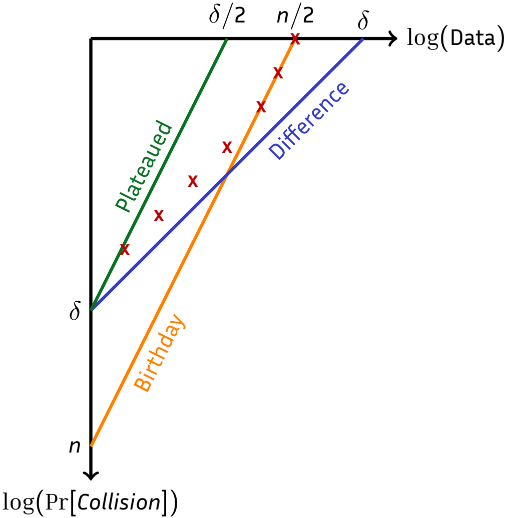

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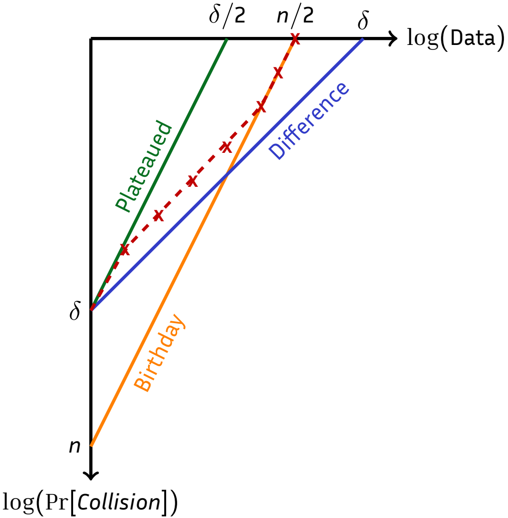
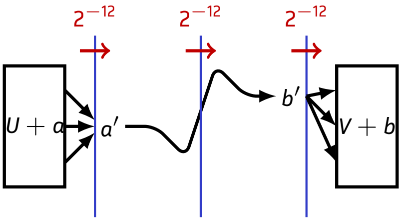


In Practice: XooDoo



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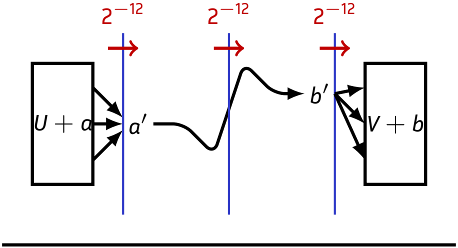

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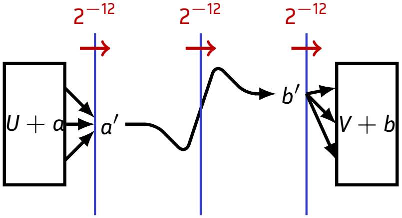

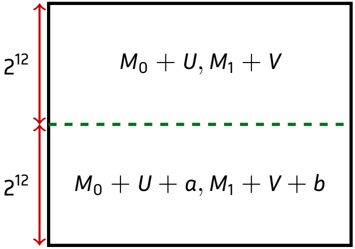

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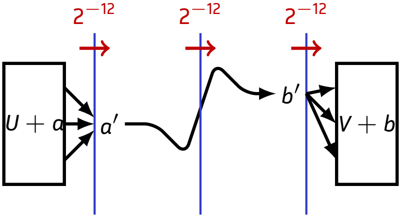

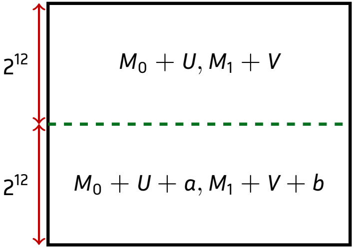

$$\text{Moy} \approx 2^{-24}$$

$$\Pr[Collision] = 2^{12} \times 2^{-24}$$

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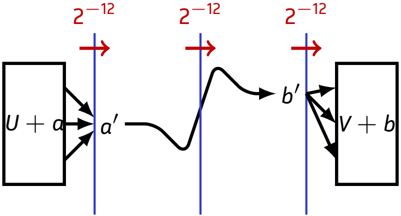

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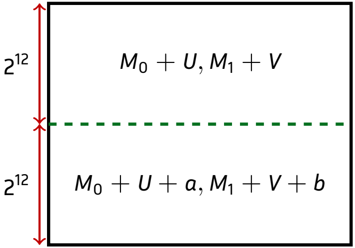

$$\Pr[Collision] = 2^{12} \times 2^{-24}$$
$$>> 2^{13} \times 2^{-36}$$

Experiments on 3-rounds XooDoo

- a' touch 6 different S-boxes;
- Apply techniques to a subspace of dimension $6 \times 3 = 18$.

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- Apply techniques to a subspace of dimension $6 \times 3 = 18$.

But,

If we obtain a collision in a set, we obtain 8 collisions.

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On the choice of the subspace of dimension 12, 3 millions random sets of size 2^{13}.

0	1	2	3	4	5	6	7
*	200	65	17	8	0	1	0

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On the choice of the subspace of dimension 12, 3 millions random sets of size 2^{13}.

0	1	2	3	4	5	6	7
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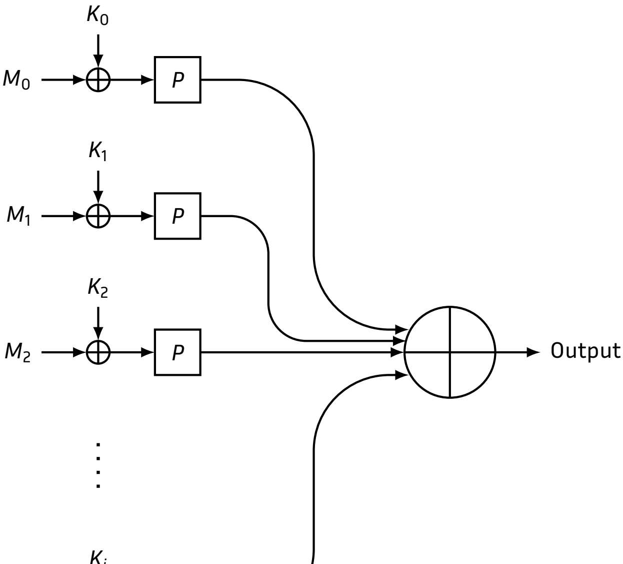
The probability of getting A collision is then smaller than expected...

Where this behaviour comes from?

Plan of this Section

New Criteria: Squared pseudo-Walsh Coefficient



Framework

- P operates on n-bit words;
- indepent keys with $|K_i| = |M_i|$;
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Goal

Find a collision in the output

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Let $M = (M_0, M_1, \ldots, M_i)$ and $M' = (M'_0, M'_1, \ldots, M'_j)$.

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$$\sum_{\alpha=0}^{i} P(M_\alpha + K_\alpha) + \sum_{\beta=0}^{j} P(M'_\beta + K_\beta) = F_j(M, M') + F_{i,j}(M)$$

where

$$F_j(M, M') = \sum_{\alpha=0}^{j} (P(M_\alpha + K_\alpha) + P(M'_\alpha + K_\alpha))$$

and

$$F_{i,j}(M) = \sum_{\beta=j+1}^{i} P(M_\beta + K_\beta)\,.$$

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and

$$F_{i,j}(M) = \sum_{\beta=j+1}^{i} P(M_\beta + K_\beta)\,.$$

Goal

$$p = \Pr\left[\sum_{\alpha=0}^{i} P(M_\alpha + K_\alpha) = \sum_{\beta=0}^{j} P(M'_\beta + K_\beta)\right]$$

New criteria

$$p = \Pr\left[F_j(M, M') + F_{i,j}(M) = 0\right]$$

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Independent keys implies

$$\Pr\left[F_{i,j}(M) = A\right] = 2^{-n},$$

$$p = 2^{-n} \sum_{A \in \mathbb{F}_2^n} \Pr\left[F_j(M, M') = A\right].$$

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$$p = 2^{-n} \sum_{A, b_1, \ldots, b_j \in \mathbb{F}_2^n} \mathrm{DP}(a_0, A + b_1 + \cdots + b_j)\mathrm{DP}(a_1, b_1)\mathrm{DP}(a_2, b_2) \cdots \mathrm{DP}(a_j, b_j).$$

Pseudo-Walsh Transform

We pose, for $a \in \mathbb{F}_2^n$,

$$\mathcal{W}_P^a(\mu) = \sum_{b \in \mathbb{F}_2^n} (-1)^{b \cdot \mu} \mathrm{DP}(a, b) .$$

Then, we have

$$\mathrm{DP}(a, b_0) = \frac{1}{2^n} \sum_{\mu \in \mathbb{F}_2^n} (-1)^{b_0 \cdot \mu} \mathcal{W}_P^a(\mu) .$$

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$$(\mathrm{DP}(a_0) * \mathrm{DP}(a_1)(b_0) = \sum_{b \in \mathbb{F}_2^n} \mathrm{DP}(a_0, b) \mathrm{DP}(a_1, b + b_0)$$

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$$(\mathrm{DP}(a_0) * \mathrm{DP}(a_1)(b_0) = \sum_{b \in \mathbb{F}_2^n} \mathrm{DP}(a_0, b)\mathrm{DP}(a_1, b + b_0)$$

- Associative
- Commutative
- Bilinear

Pseudo-Walsh Tranform

$$p = \mathcal{W}_P^{-1}\left(\mathcal{W}_P[\mathrm{DP}(a_0)](\mu)\mathcal{W}_P[\mathrm{DP}(a_1)](\mu)\cdots\mathcal{W}_P[\mathrm{DP}(a_j)](\mu)\mathcal{W}_P[\mathrm{DP}(uni)](\mu)\right)(0)$$

which can be expressed with

$$p = \frac{1}{2^n}\sum_{\mu\in\mathbb{F}_2^n}\mathcal{W}_P[\mathrm{DP}(a_1)](\mu)\cdots\mathcal{W}_P[\mathrm{DP}(a_j)](\mu)\mathcal{W}_P[\mathrm{DP}(uni)](\mu)$$

$$\sum_b \mathrm{DP}(a_i, b) = 1\,.$$

This means then exactly that for all $\mu \in \mathbb{F}_2^n$ and for all a_i, $|\mathcal{W}_P[\mathrm{DP}(a_i)](\mu)| \leq 1$.

Pseudo-Walsh Transform

Moreover, as $\mathrm{DP}(uni) = 2^{-n}$, we have $\mathcal{W}_P[\mathrm{DP}(uni)](\mu) = 0$ for all $\mu \neq 0$ and $\mathcal{W}_P[\mathrm{DP}(uni)](0) = 1$. This $\mathcal{W}_P[\mathrm{DP}(uni)]$ appears if and only if the size of the messages are different. If this is the case, we obtain the probability

$$p = \frac{1}{2^n}\mathcal{W}_P[\mathrm{DP}(a_1)](0)\mathcal{W}_P[\mathrm{DP}(a_2)](0)\cdots\mathcal{W}_P[\mathrm{DP}(a_j)](0)$$

but for all 2^n differential vector,

$$\mathcal{W}_P[\mathrm{DP}(a_2)](0) = \sum_{b\in\mathbb{F}_2^n} \mathrm{DP}(a, b) = 1$$

Hence, when two messages are of different size, the probability that guetting a collision is exactly 2^{-n}.

Best choice is when $a_1 = a_2$

$$\sum_{b \in \mathbb{F}_2^n} \mathrm{DP}(a_0, b)\mathrm{DP}(a_1, b) = \frac{1}{2}\left(\sum_{b \in \mathbb{F}_2^n} \mathrm{DP}(a_0, b)^2 + \mathrm{DP}(a_1, b)^2\right)$$

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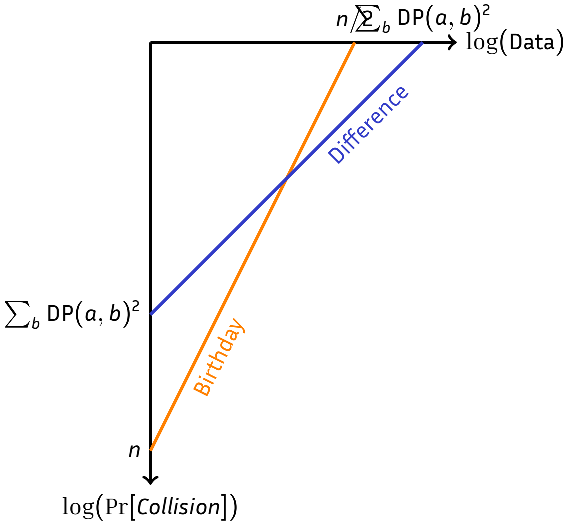
So we focus on

$$\max_{a \in \mathbb{F}_2^n} \sum_{\in \mathbb{F}_2^n} \mathrm{DP}(a, b)^2$$

Tight or not tight?



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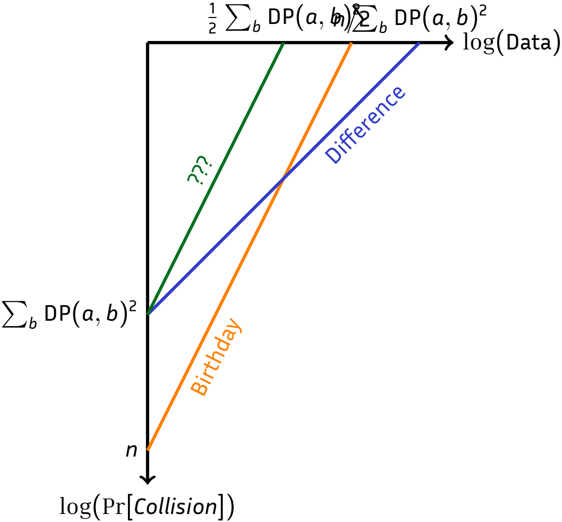

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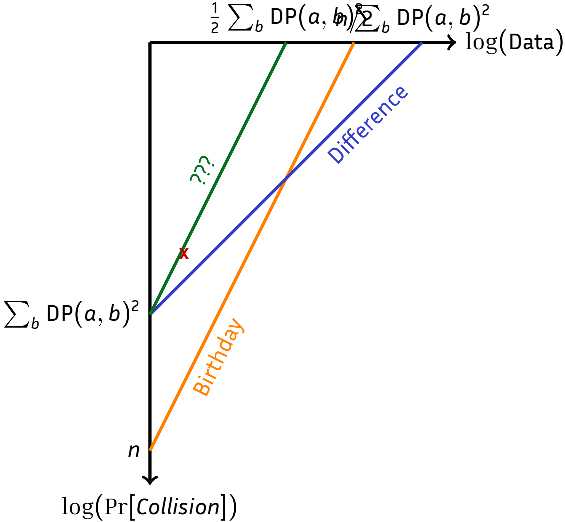

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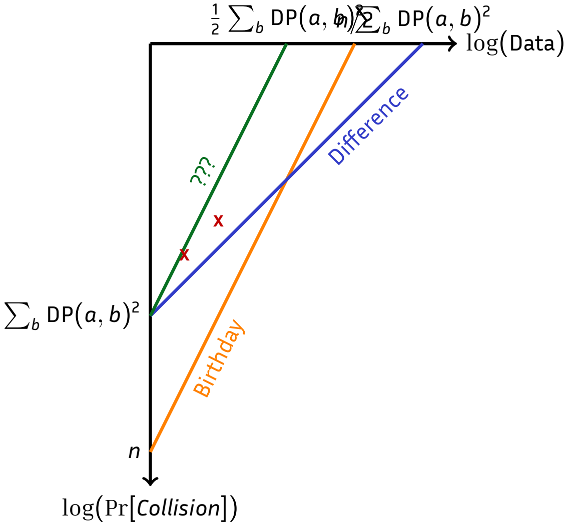

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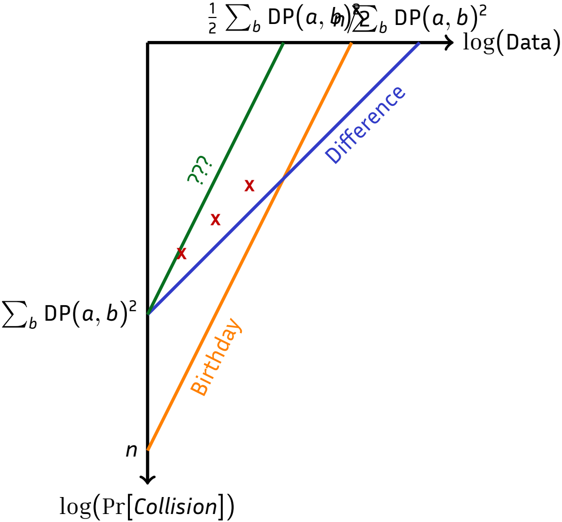

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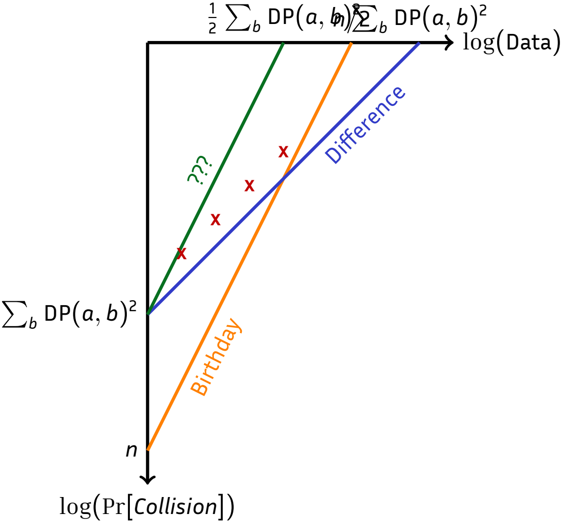

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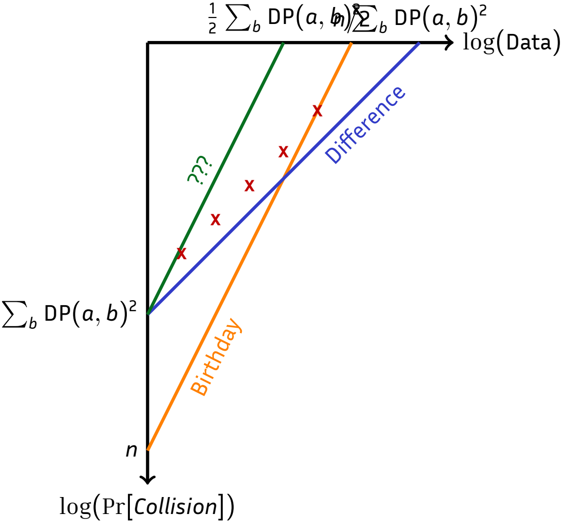

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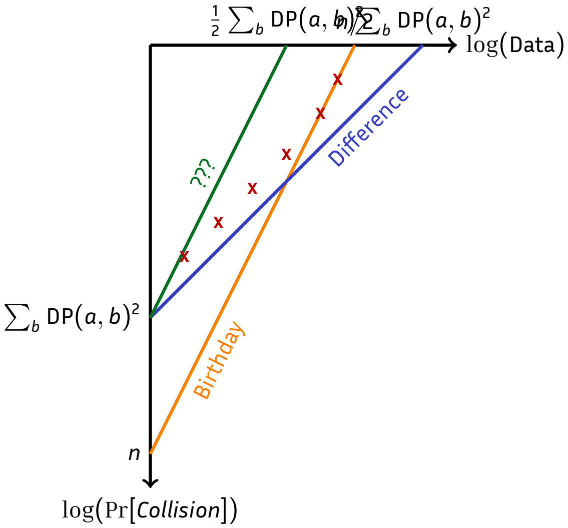

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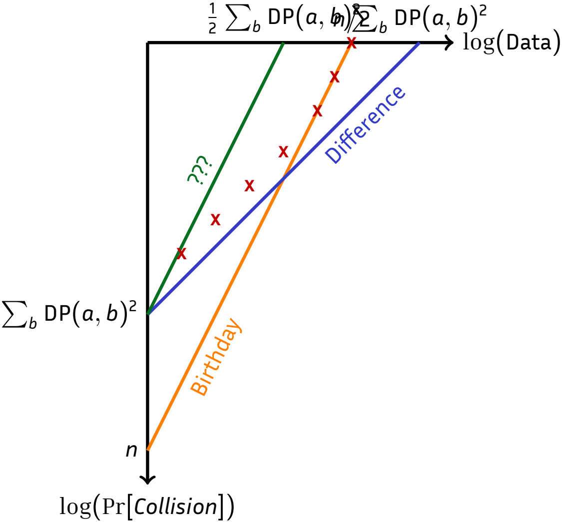

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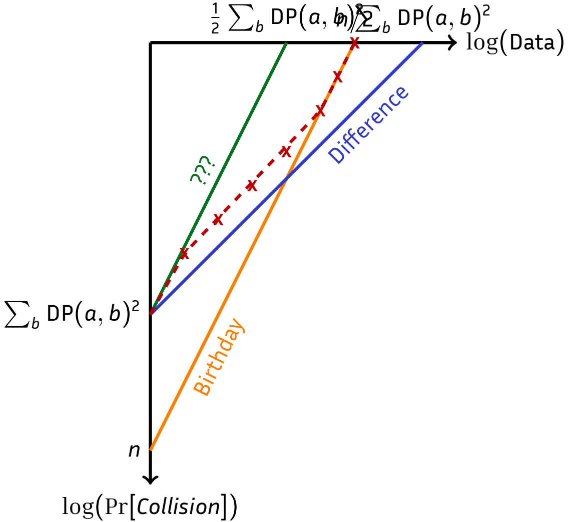

Tight in number of queries

Let Δ such that $\sum_b \mathrm{DP}(\Delta, b)^2$ is maxmal.

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$M_0 + \Delta$	$M_0 + \Delta$	$M_0 + \Delta$	$M_0 + \Delta$	$M_0 + \Delta$	$M_0 + \Delta$	$M_0 + \Delta$
$M_1 + \Delta$	M_1	M_1	M_1	M_1	M_1	M_1
M_2	$M_2 + \Delta$	M_2	M_2	M_2	M_2	M_2
M_3	M_3	$M_3 + \Delta$	M_3	M_3	M_3	M_3
M_4	M_4	M_4	$M_4 + \Delta$	M_4	M_4	M_4
M_5	M_5	M_5	M_5	$M_5 + \Delta$	M_5	M_5
M_6	M_6	M_6	M_6	M_6	$M_6 + \Delta$	M_6
M_7	M_7	M_7	M_7	M_7	M_7	$M_7 + \Delta$
M_8	M_8	M_8	M_8	M_8	M_8	M_8
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Tight in number of blocks?

- The same technique as the parallel one can be applied (win one round)
- Find a vector space... You know the rest

Using the average and not the max

$$M_0 + \Delta_0, M_1 + \Delta_0$$
$$M_0 + \Delta_1, M_1 + \Delta_1$$
$$M_0 + \Delta_2, M_1 + \Delta_2$$
$$M_0 + \Delta_3, M_1 + \Delta_3$$
$$M_0 + \Delta_4, M_1 + \Delta_4$$
$$M_0 + \Delta_5, M_1 + \Delta_5$$
$$M_0 + \Delta_6, M_1 + \Delta_6$$

$$\Pr[Collision] = \text{Moy}(\sum \delta^2) \times D^2$$

Using the average and not the max

$$M_0 + \Delta_0, M_1 + \Delta_0$$
$$M_0 + \Delta_1, M_1 + \Delta_1$$
$$M_0 + \Delta_2, M_1 + \Delta_2$$
$$M_0 + \Delta_3, M_1 + \Delta_3$$
$$M_0 + \Delta_4, M_1 + \Delta_4$$
$$M_0 + \Delta_5, M_1 + \Delta_5$$
$$M_0 + \Delta_6, M_1 + \Delta_6$$

$$\Pr[Collision] = \mathrm{Moy}(\sum \delta^2) \times D^2$$

$$\mathrm{Moy}(\sum \mathrm{DP}^2) > \frac{1}{2^n}$$

Plan of this Section

Comparisons

Parallel	Serial
2 blocks is "easier"	2 blocks is the best
DP	$\sum \mathrm{DP}^2$
bound	bound
not tight if cheat	tight if cheat
almost tight for D small	almost tight for D small

Questions

- Experiments for 3-rounds XooDoo parallel ?
- Find greater vector spaces ?
- DP is easy, but what about $\sum \mathrm{DP}^2$?