# Des nouvelles attaques sur les registres filtrés exploitant la structure des corps finis

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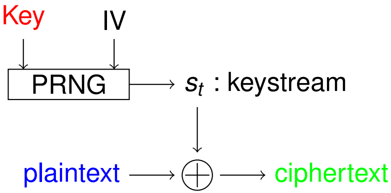
# Stream ciphers

- Symetric cryptography, $\neq$ block ciphers
- Based on Vernam cipher (one-time pad)
- PRNG



# Stream ciphers

- Block cipher modes of operations (OFB, Counter)
- Specific design (LFSR, NLFSR)
- Internal state
- Large period
- A5/1 - A5/2, SNOW

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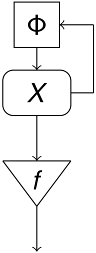
## Interests

- Small latency
- No padding
- No error propagation
- Cheap

# Generic attacks




- Key recovering

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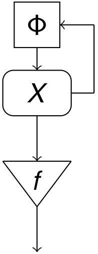


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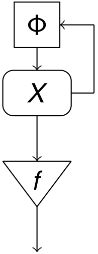


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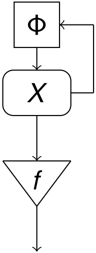


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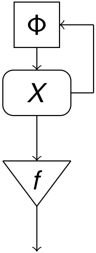


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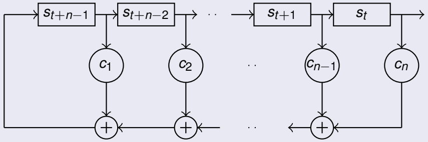
**Always take an internal state twice bigger as the security level (i.e. key size)**

# Linear feedback shift Register (LFSR)

## Definition

*Fibonacci representation*



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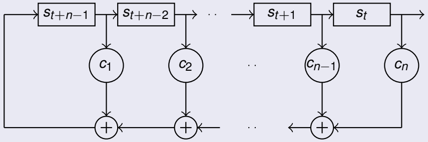
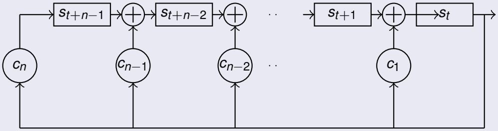



## Definition

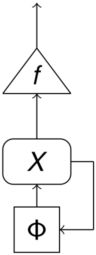
*Gallois representation*

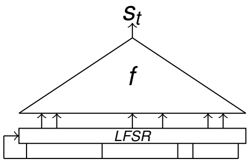


# Classical properties of LFSR

- Nice statistical properties
- Linear
- $s_{t+L} = \sum_{i=1}^{n} c_i s_{t+n-i}, \forall t \leq 0$
- $P(X) = 1 - \sum_{i=1}^{n} c_i X^i$
- $P^*(X) = X^n P(1/X)$
- We wil take $P$ primitive

# Filtered LFSR




$$s_t = f(u_{t+\gamma_1}, \cdots, u_{t+\gamma_n})$$

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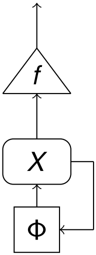


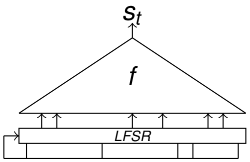


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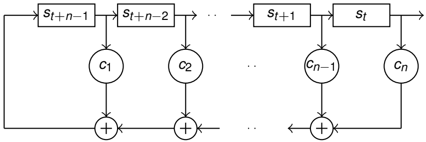
## Algebraic Normal Form

$$f(x_1, x_2, \cdots, x_n) = \sum_{u \in \mathbb{F}_2^n} a_u \prod_{i=1}^{n} x_i^{u_i}$$

$$= a_0 + a_1 x_1 + a_2 x_2 + \cdots + a_3 x_1 x_2 + \cdots + a_{2^n-1} x_1 \cdots x_n$$

# LFSR over a Finite Field

- $\alpha$ : root of the primitive characteristic polynomial in $\mathbb{F}_{2^n}$
- Identify the $n$-bit words with elements of $\mathbb{F}_{2^n}$ with the dual basis of $\{1, \alpha, \alpha^2, \cdots, \alpha^{n-1}\}$

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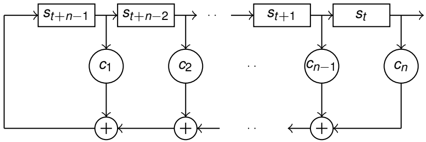


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**For all** $t$, $X_t = X_0\alpha^t$

# Boolean functions

**Proposition (Univariate representation)**

$$F(X) = \sum_{i=0}^{2^n-1} A_i X^i$$

*with $A_i \in \mathbb{F}_{2^n}$ given by the discrete Fourier Transform of $F$*

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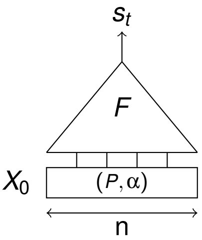
# Monomial equivalence [Rønjom - Cid 2010]

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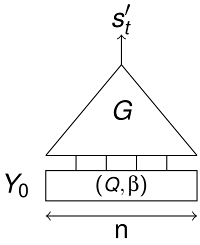


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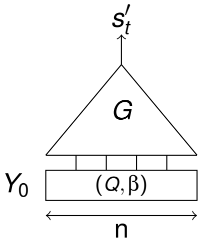


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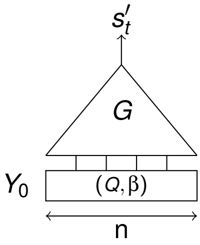


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**If** $G(x) = F(x^r)$

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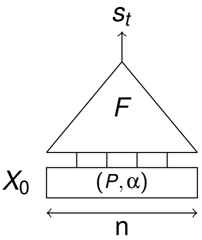


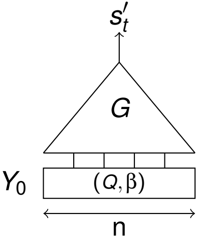


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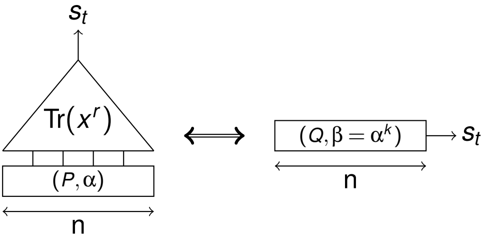
**For all $t$, $s'_t = s_t$ if $Y_0 = X_0^k$**

# Example

$F(x) = \mathrm{Tr}(x^r)$, with $\gcd(r, 2^n - 1) = 1$ :
Let $k$ be such that $rk \equiv 1 \mod (2^n - 1)$.




$\Longrightarrow$ The initial generator is equivalent to a plain LFSR of the same size.

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- Algebraic attacks
- Correlation attacks

# Algebraic attacks

$\Lambda$ : Linear complexity

**Proposition (Massey-Serconek 94)**

*Let an LFSR of size n filtered by a Boolean function F :*

$$F(X) = \sum_{i=0}^{2^n-1} A_i X^i$$

*Then*

$$\Lambda = \#\{0 \leq i \leq 2^n - 2 : A_i \neq 0\}$$

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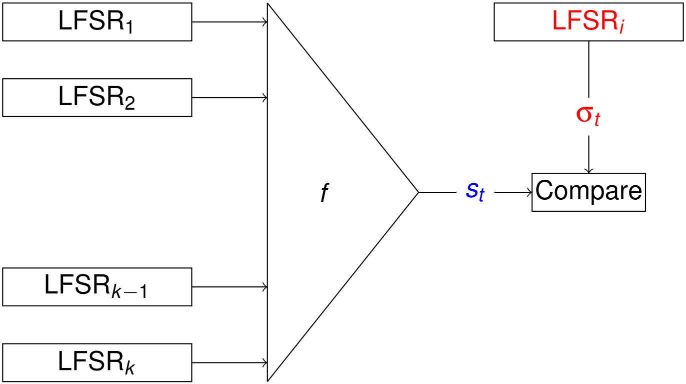
**The monomial equivalence does not affect the complexity of algebraic attacks [Gong et al. 11]**

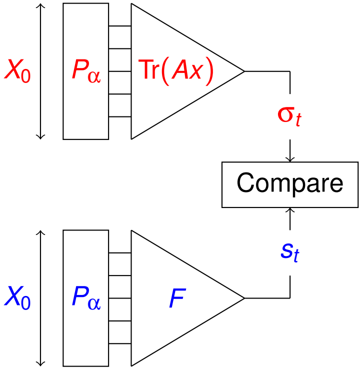
# Correlation attack [Siegenthaler 85]



# Criterion

The criterion besides the correlation attack is the **resiliency**.

# Fast correlation attack [Meier - Staffelbach 88]

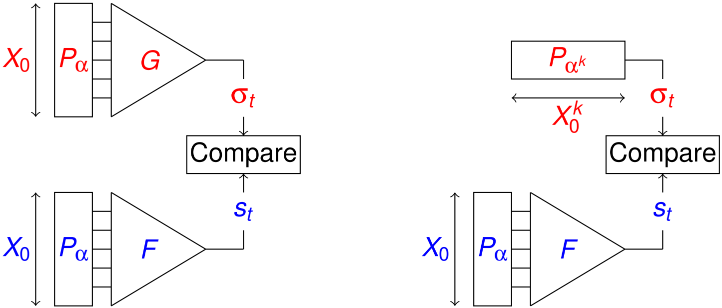


# Criterion

The criterion besides the fast correlation attack is the **non-linearity**.

# Generalized fast correlation attacks

$$G(x) = \mathrm{Tr}(Ax^k)$$



# Generalized non-linearity [Gong & Youssef 01]

Relevant security criterion :

## Generalized non-linearity

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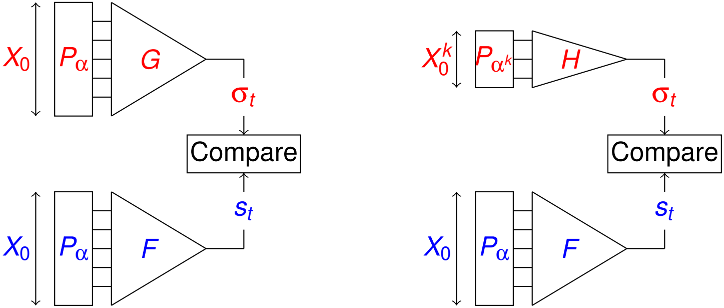
**And if $k$ is not coprime to $2^n - 1$ ?**

# A more efficient correlation attack

When $\gcd(k, 2^n - 1) > 1$ and $F$ correlated to $G(X) = H(X^k)$.



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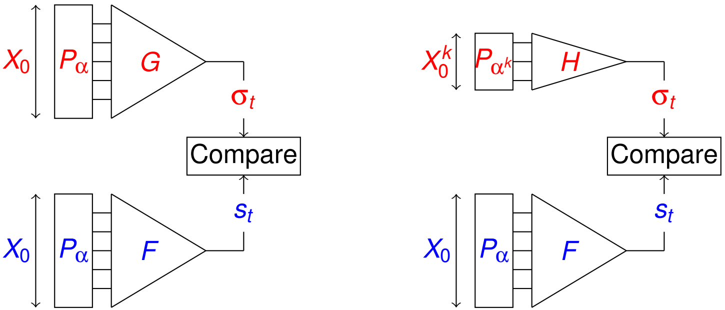


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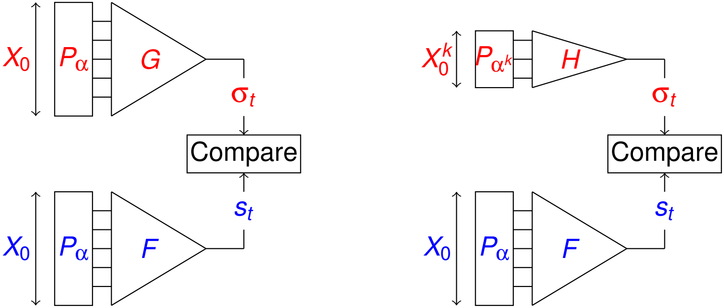


- Number of states of the small generator : $\tau_k = \mathrm{ord}(\alpha^k)$.
- Exhaustive search on $X_0^k$ : Time $= \frac{\tau_k \log(\tau_k)}{\varepsilon^2}$

# Recovering the remaining bits of the initial state

**Property**

We get $\log_2(\tau_k)$ bits of information on $X_0$ where $\tau_k = \mathrm{ord}(\alpha^k)$ :

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If we perform two distinct correlation attacks with $k_1$ et $k_2$, then we get $\log_2(\mathrm{lcm}(\tau_{k_1}, \tau_{k_2}))$ bits of information.

# First improvement

The complexity

$$\text{Time} = \frac{\tau_k \log(\tau_k)}{\varepsilon^2}$$

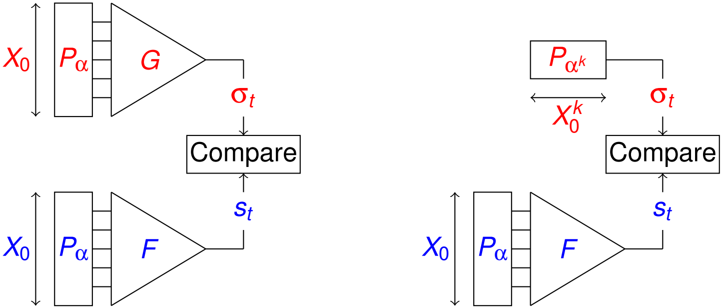
can be reduced to

$$\text{Time} = \tau_k \log \tau_k + \frac{2 \log(\tau_k)}{\varepsilon^2} \; .$$

with a fast Fourier transform [Canteaut - Naya-Plasencia 2012]

# Second improvement

$G(X) = H(X^k)$ when $H$ is linear :




- Size of the small LFSR : $L(k) = \mathrm{ord}(2) \mod \tau_k$.
- If $L(k) < n$ and $H$ is linear $\longrightarrow$ fast correlation attack.

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- Split the state on the multiplicative subgroups
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**What is the generalization ?**

Do we generalize the **resiliency** ?

# Conclusion and open questions

## Conclusion

- Generalized criterion for $f$ besides the generalized non-linearity.
- The attack does not apply when $(2^n - 1)$ is prime.

## Open questions

- Find good filtering Boolean functions ?
- Compute efficiently a good approximation of the filtering function ?

**Thank You for your attention !**

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**Questions ?**