

# GENERIC ATTACKS ON DUPLEX-BASED AEAD MODES

SMALL CYCLES AND LARGE COMPONENTS

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# GRAPH OF FUNCTION

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We call  $\mu(x)$  and  $\lambda(x)$  the cycle length and tail length respectively

# RELEVANT VALUES

## DEFINITION ( $\nu$ -COMPONENT)

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## DEFINITION ( $(s, \nu)$ -COMPONENT)

let  $0 < \nu < \frac{1}{2}$  and  $0 < s < 1$ . A  $(s, \nu)$ -component is a component whose size is **greater or equal to**  $ns$  and whose cycle is of size **at most**  $n^{\frac{1}{2}-\nu}$ .

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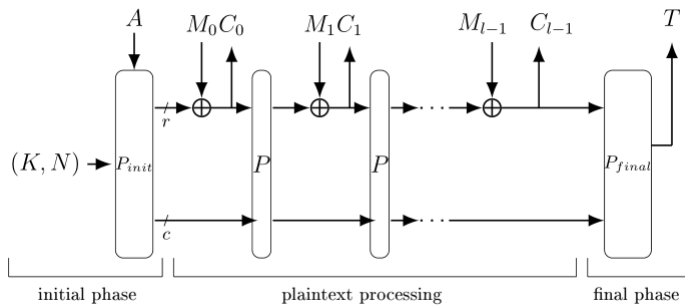
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De Laurentis, Crypto 1987, "Components and Cycles of a random function"

# DUPLEX AEAD



# SECURITY OF DUPLEX

**Simplified** Beyond conventional security in sponge-based authenticated encryption modes [Jovanovic, Luykx, Mennink, Sasaki, Yasuda, JoC 2019]

$$\mathcal{T} \ll \min\left\{2^{\frac{b}{2}}, \frac{2^c}{\alpha}, 2^k\right\} \text{ and } q_d \ll 2^\tau$$

where,  $\alpha < r$ , where  $q_d$  is the number of forgery attempts.

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So duplex construction is proven for  $2^{\frac{c}{2}}$ , and known generic attacks are in  $\frac{2^c}{\alpha}$

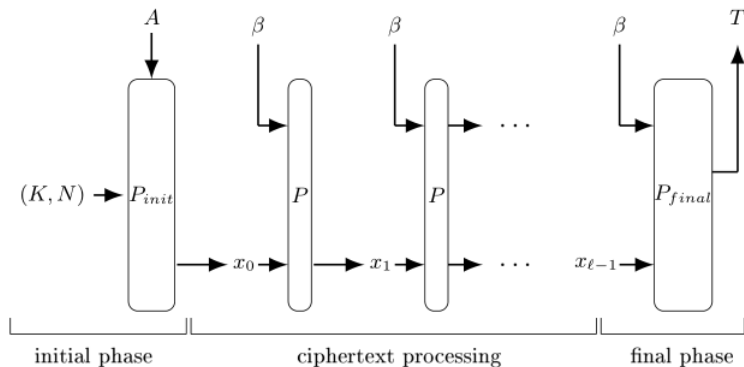
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Let  $C_\beta^\ell = \beta_\ell = \underbrace{\beta || \dots || \beta}_\ell$ . Then the decryption of  $C_\beta^\ell$  corresponds to the iteration of

$$f_\beta : \mathbb{F}_2^c \longrightarrow \mathbb{F}_2^c \\ x \longmapsto [P(\beta || x)]_c.$$





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**Online** : input  $(N, A, C, T)$  with  $N, A$  possibly different and  $C = C_\beta^\ell$  with  $\ell = \gamma 2^{\frac{c}{2}}$ . And  $T$  being derived from a value of the cycle of  $f_\beta$  ( $n^{\frac{1}{2}-v}$  possibilities at most)

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**Complexity**  $O\left(2^{\frac{3c}{4}}\right)$

# EXPERIMENTAL VERIFICATION

Statistics verified up to small  $c$  values.

## SPECIFIC MODES AND PADDING

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- ▶ Motorist : Keyak

# WHAT FRUSTRATES THE ATTACK

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- ▶ Use a  $\rho$ -like application (Beetle, Subterranean)