

# How to use differential trails to attack compression functions

Joan Daemen, Jonhathan Fuchs and Yann Rotella  
Dagstuhl, Germany

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# Structure of this Talk

- 1 Introduction
- 2 Serial Construction
- 3 Parallel Construction
- 4 Conclusion

## The Serial Construction

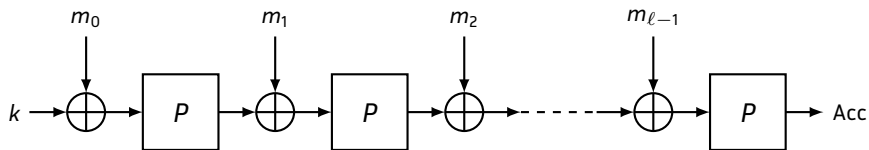


Figure: The Serial Construction

## Parallel Construction

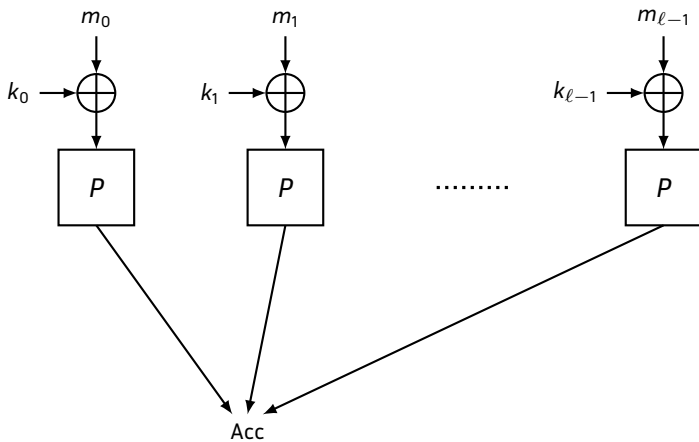
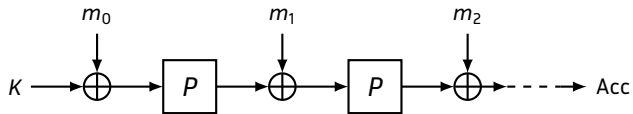
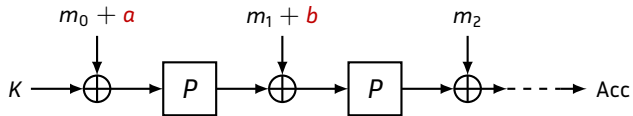


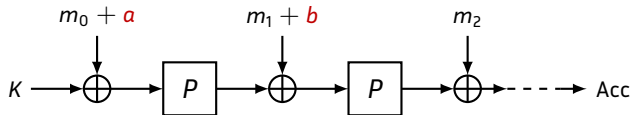
Figure: The Parallel Construction

## Plan of this Section

- 1 Introduction
- 2 **Serial Construction**
  - Very known facts
  - Real Attack
- 3 Parallel Construction
- 4 Conclusion



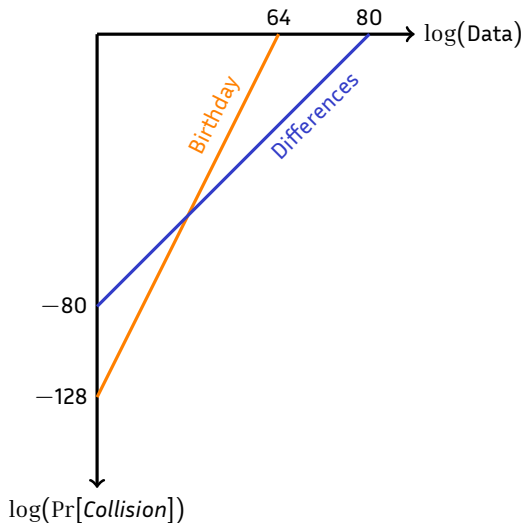




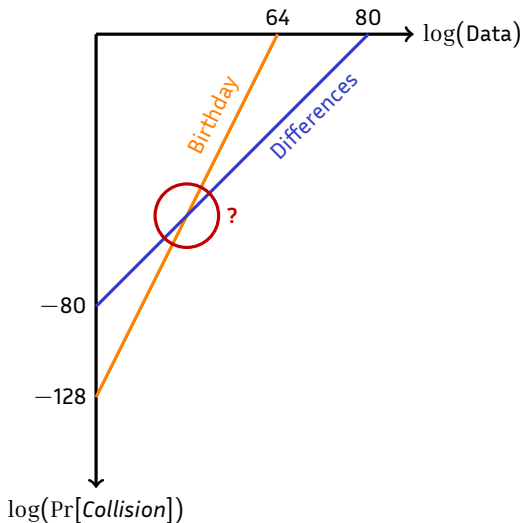
$$\Pr[\text{Collision}] = DP(a, b)$$



## Birthday VS Difference



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## Using Covering Vector spaces

$\langle (a_1, b_1), (a_2, b_2), \dots, (a_v, b_v) \rangle = V$  such that

$$\sum_{(a,b) \in V} \delta_{a,b} > \delta.$$

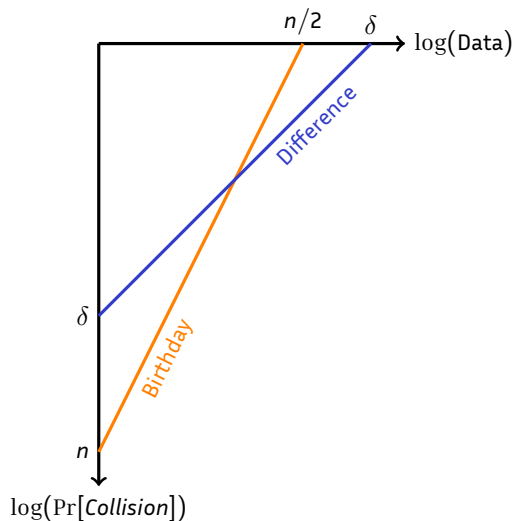
By making this strategy:

$$\begin{array}{c}
 M_0, M_1 \\
 M_0 + a_1, M_1 + b_1 \\
 M_0 + a_2, M_1 + b_2 \\
 M_0 + a_1 + a_2, M_1 + b_1 + b_2 \\
 \vdots \\
 M_0 + \sum a_i, M_1 + \sum b_i
 \end{array}$$

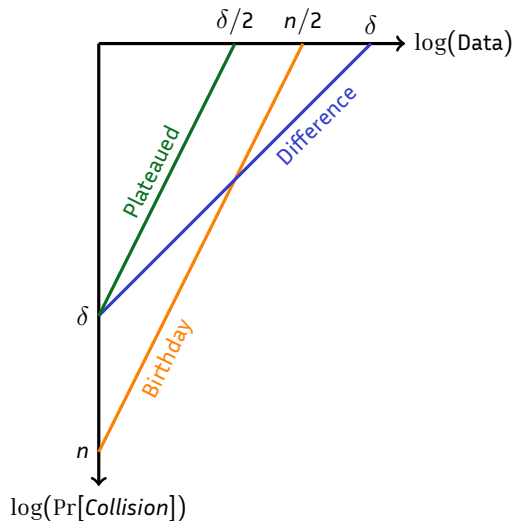
$$\begin{array}{c}
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 \vdots \\
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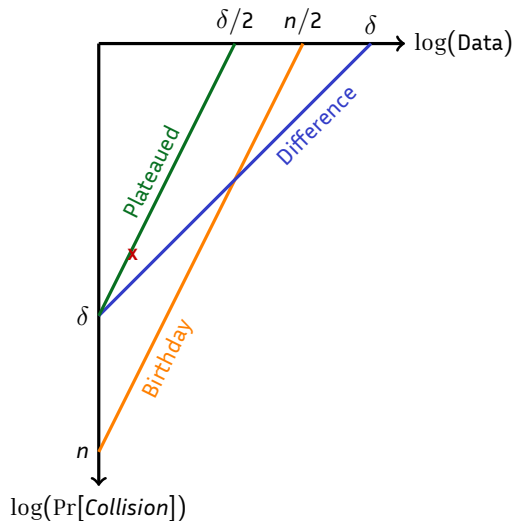
## In terms of Security



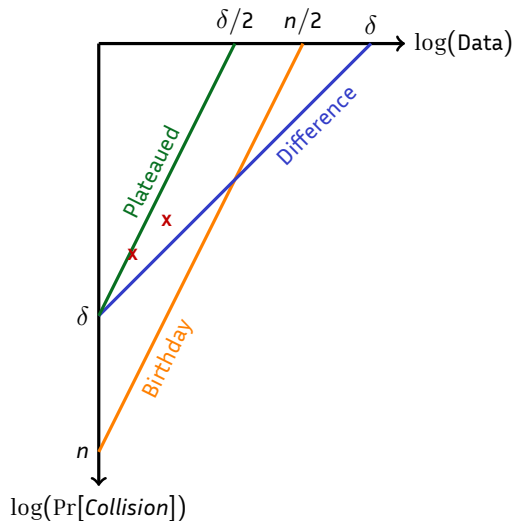
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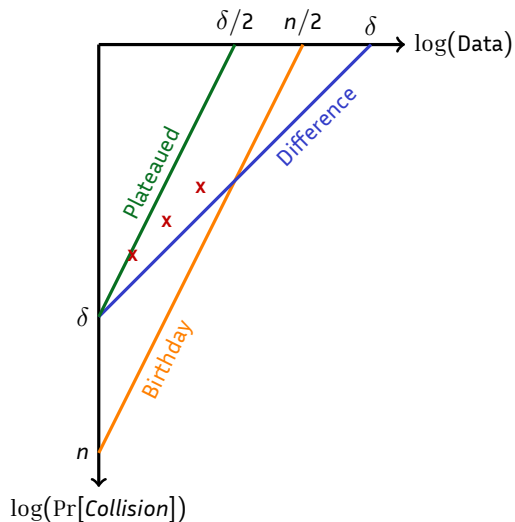
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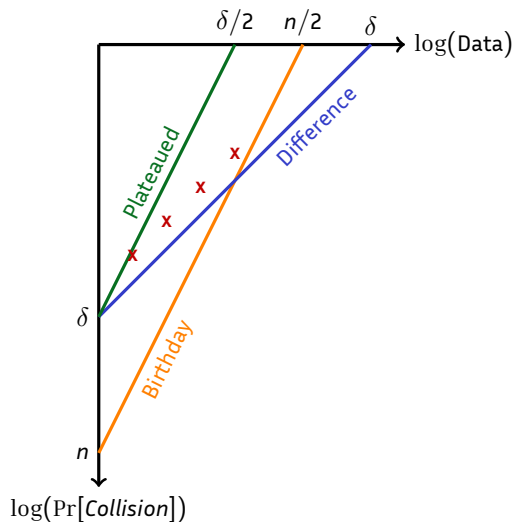


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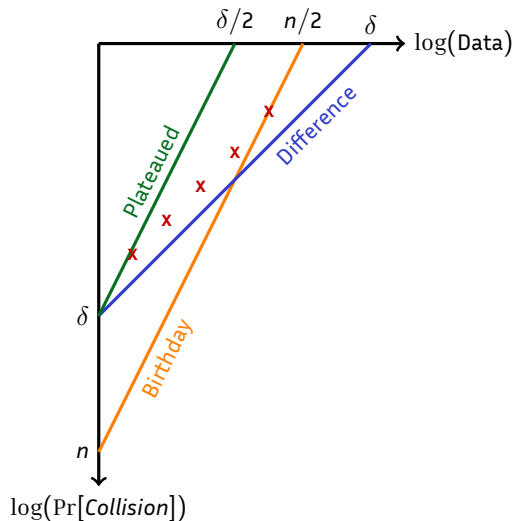




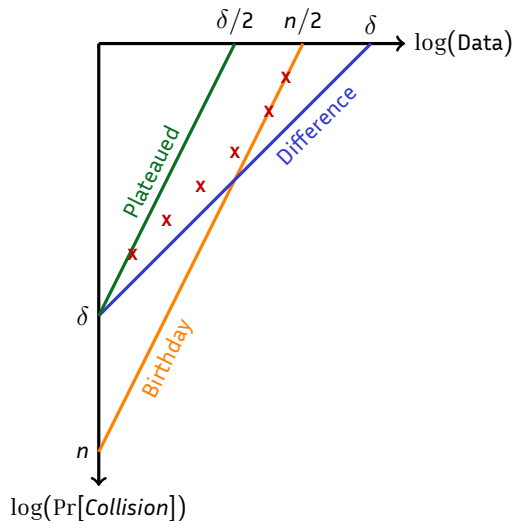
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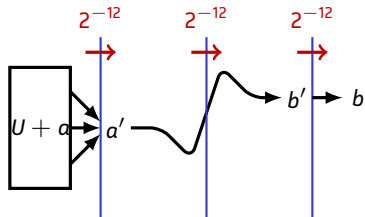
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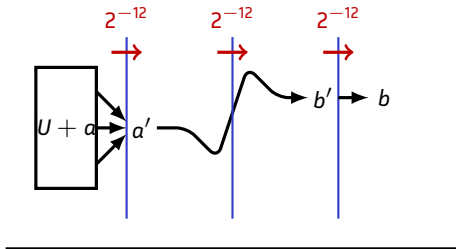
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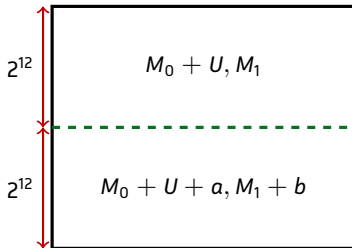
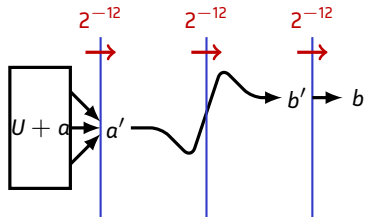
## In Practice: XooDoo



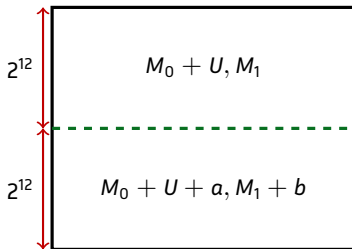
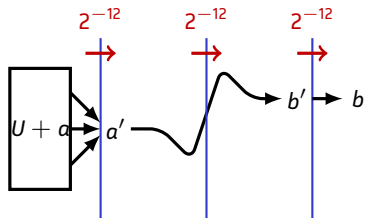
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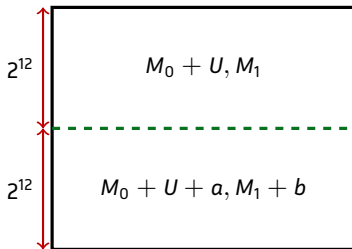
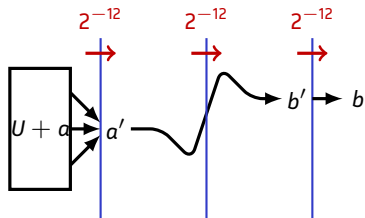
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Number of pairs st  $a'$ :  $2^{12}$

$$\Pr[\text{Collision}] = 2^{12} \times 2^{-24}$$

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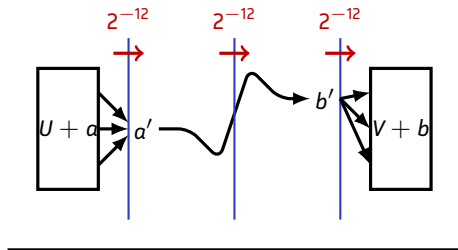


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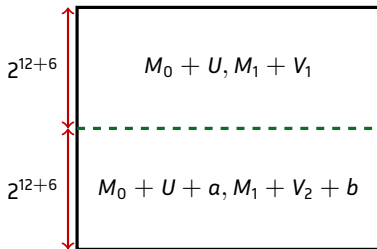
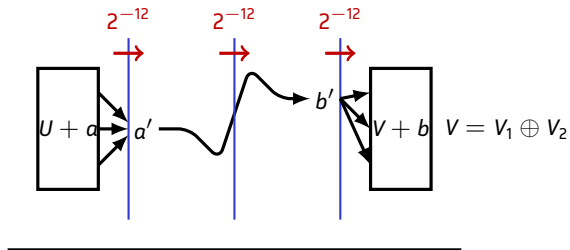
$$\Pr[\text{Collision}] = 2^{12} \times 2^{-24} \\ = 2^{-12}$$



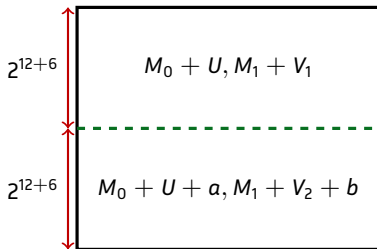
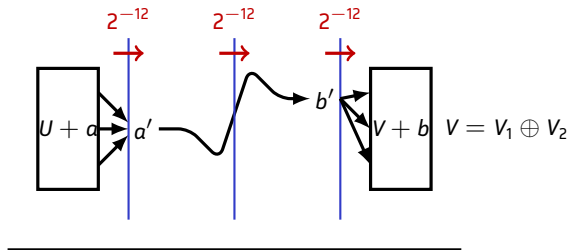
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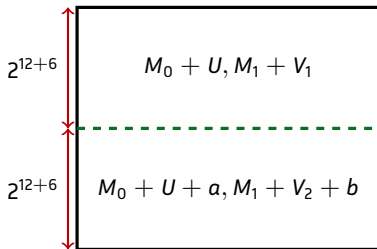
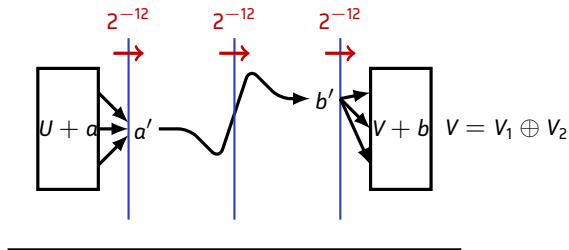
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 Win wp. 1 with  $2^{19}$ .

## Security Criteria

If trail  $a \mapsto b$  with probability  $2^{-w_1-w_2-w_3 \cdots -w_r}$ , we get collision with probability

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$$2^{w_1-w_2-w_3-\dots-w_{r-1}}$$

using

$$D = 2^{1+w_1+w_r/2}$$

We gain the first round and the half of the last round

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## New Criteria: Squared pseudo-Walsh Coefficient

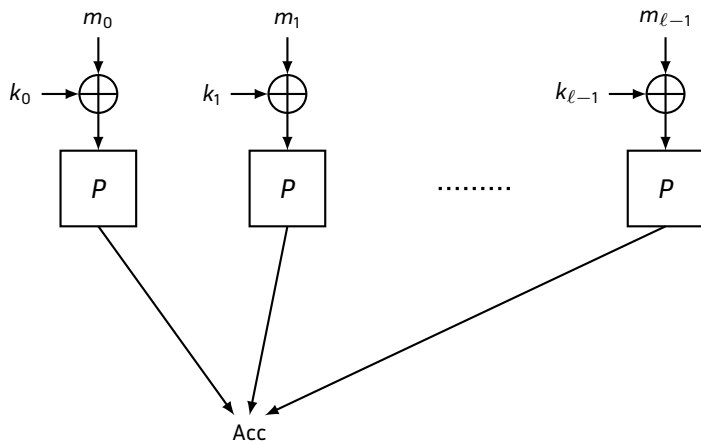


Figure: The Parallel Construction



## Results

If Keys are independent and uniformly distributed, then

$$\Pr[F(M) = F(M') | M + M' = \Delta]$$

is maximal when  $\Delta$  has the same value on two blocks exactly.

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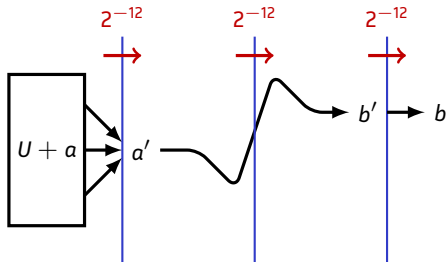
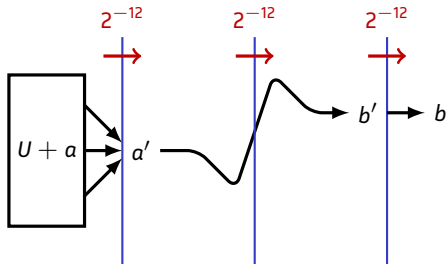
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The relevant criteria is

$$\max_a \sum_b (DP(a, b))^2$$

## In iterated construction



## Security Criteria

- Complexity:  $2^{2w_1+2w_2+\dots+2w_{r-1}+w_r}$ .

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# Conclusion

Both strategies share the same security criteria:

- The first round doesn't count;
- The last round counts for half.

But...

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- The first round doesn't count;
- The last round counts for half.

But... The parallel strategy seems to offer twice the security.